HEAT AND MASS TRANSFER IN THE FREE-MOLECULAR MOTION OF A GAS IN A CHANNEL OF FINITE LENGTH

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This article examines the solution of the problem of heat and mass transfer in the Knudsen flow of a gas in a channel of finite dimensions with boundary conditions which permit the use of different models of the core of gas scattering by the channel surface.

The recent literature mainly contains only methods of solving isothermal problems of gas flow in channels of finite length based on the assumption of diffuse scattering of gas molecules by the channel walls. The exceptions are the studies [1, 2]. De Marcus [1] found an isothermal gas flow rate in a finite channel with mirror-diffuse scattering of molecules on the walls. However, within the framework of the method of calculation proposed by the author, it is unclear both how the temperature gradient is to be accounted for and how more general laws of scattering of gas molecules by the channel surface are to be established. The study [2] obtained a numerical solution to an isothermal problem with incomplete accommodation of the tangential component of momentum. However, the authors had to assign the distribution function of the reflected molecules in the form of an exponential series in velocities. This approach to formulation of the boundary conditions is insufficiently rigorous.

The goal of the present study is to calculate the parameters of the nonisothermal motion of a low-density gas in a channel of finite length with a formulation of the boundary conditions that will permit the use of different models of the core of gas scattering by the surface. We chose the mirror-diffuse model [3] and the model of Cercignani and Lampis [4] to perform specific calculations.

1. We are examining the free-molecular steady flow of a monatomic gas in a cylindrical channel of radius R and length L connecting two volumes. The inside surface of the channel is isotropic. The gas in the volumes is in equilibrium states described by Maxwell distributions f_1° and f_2° (see Fig. 1). The relative pressure and temperature gradients at the ends of the channel are assumed to be small. As the scales of measurement of length, temperature, pressure, velocity, the distributions, and the numerical densities we respectively chose the quantities 2R, T_1, P_1, $\beta^{-1/2} = (2kT_1/m)^{1/2}$, $p_1\beta^{3/2}/kT_1$, n_1, where T_1, P_1, n_1 are the temperature, pressure, and numerical density in the left volume.

Since the determining process in the free-molecular flow regime is the interaction of the gas with the channel walls, then to calculate the macroscopic characteristics of the gas flow it is sufficient to find the distribution of the molecules reflected from the surface of the channel $f^+(z, \mathbf{c})$. The boundary condition linking the distribution of molecules incident on the wall and the distribution of molecules reflected by the wall has the form [5]

$$|c_n| f^+(z, \mathbf{c}) = \int\limits_{c_n'<0} |c_n'| f^-(z, \mathbf{c}') W(z, \mathbf{c}' \to \mathbf{c}) d\mathbf{c}'. \tag{1}$$

The assumption of the smallness of the relative pressure and temperature gradients makes it possible to linearize the problem, for example, about the equilibrium state of the gas in the left vessel. Then we can write the following for the functions $f_2^\circ(c)$ and $f^+(z, \mathbf{c})$:

$$f_2^0(c) = f_1^0(c)\left\{1 + l\left[\nu + \tau\left(c^2 - \frac{5}{2}\right)\right]\right\},$$

$$f^+(z, \mathbf{c}) = f_1^0\left\{1 + z\left[\nu + \tau\left(c^2 - \frac{5}{2}\right)\right] + h^+(z, \mathbf{c})\right\}, \tag{2}$$

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where $\nu = \Delta P/l$, $\tau = \Delta T/l$, $h^+(z, \mathbf{c})$ is the perturbation function. Meanwhile, it is assumed that $h^+ \ll 1$.

2. In the free-molecular regime of gas flow, the molecule distribution does not change along the path of molecular motion. Thus, proceeding on the basis of the geometry of the problem, we can represent the function $f^-(z, \mathbf{c})$ at an arbitrary point on the channel wall in the form:

$$f^-(z, \mathbf{c}_R) = f_1^0\theta\left(c_z - \frac{z}{h_0}\right) + f^+(z - h_0c_z, \mathbf{c})\theta\left(c_z - \frac{z-l}{h_0}\right)\theta\left(\frac{z}{h_0} - c_z\right) + f_2^0\theta\left(-\frac{z-l}{h_0} - c_z\right), \quad 0 \leqslant z \leqslant l, \tag{3}$$

where $h_0 = c_n/(c_n^2 + c_\varphi^2)$, $\mathbf{c}_R = (-c_n, c_\varphi, c_z)$. Equation (3) reflects the fact that the molecules, with the velocity $\mathbf{c}_R$, arrive at point z either from the ends of the channel or from its lateral surface. As a result of insertion of Eqs. (2) and (3) into boundary condition (1) and allowance for the reciprocity relation and the normalization condition for the scattering core [5], we find an integral equation for the perturbation function in the case of a linear distribution of channel-wall temperature along the z axis:

$$\begin{aligned} h^+(z, \mathbf{c}) = &\int\limits_{c_n'>0} d\mathbf{c}' W_1(\mathbf{c}_R \to \mathbf{c}')\Big\{h^+(z - h_0'c_z', \mathbf{c}')\theta\left(c_z' - \frac{z-l}{h_0'}\right)\theta\left(\frac{z}{h_0'} - c_z'\right) + \\ &+\left[\nu + \tau\left(c^2 - \frac{5}{2}\right)\right]\left[(z - h_0'c_z')\theta\left(c_z' - \frac{z-l}{h_0'}\right)\theta\left(\frac{z}{h_0'} - c_z'\right) + l\theta\left(-\frac{z-l}{h_0'} - c_z'\right) - z\right]\Big\}, \end{aligned} \tag{4}$$

where $W_1(\mathbf{c}' \to \mathbf{c}) = W(0, \mathbf{c}' \to \mathbf{c})$.

3. It follows from integral Eq. (4), written with a mirror-diffusion model of the scattering core, that the sought function can be represented in the form

$$h^+(z, \mathbf{c}) = \psi(z) + F(z, \mathbf{c}).$$

In this case, the equation for $h^+(z, \mathbf{c})$ breaks down into a system of two integral equations for the functions $F(z, \mathbf{c})$ and $\psi(z)$. The equation for $F(z, \mathbf{c})$ is solved by the method of successive approximations with the assumption that the function $\psi(x)$ is known. Insertion of the expression found for $F(z, \mathbf{c})$ into the second equation of the system leads to a second-order Fredholm equation which is a generalization of the Clausing equation. It is solved by the Ritz variational method [6]. The convergence of this method was examined in [7], where it was shown that the solutions of the Clausing equation found in the first and second approximations differ by no more than 0.1%. It follows from analysis of the integral equation for $\psi(z)$ that it can be approximated by the function

$$\psi(z) = A\varepsilon\left(\nu - \frac{\tau}{2}\right)\left(\frac{l}{2} - z\right),$$

where A is the variational constant. Calculations lead to the following form of the perturbation function for mirror-diffuse boundary conditions:

$$\begin{aligned} h^+(z, \mathbf{c}) = \sum_{n=0}^{\infty}(1-\varepsilon)^n\Big\{&\Big[A\varepsilon\left(\nu - \frac{\tau}{2}\right)\left(\frac{l}{2} - z + nh_0c_z\right) + \\ &+\left[\nu + \tau\left(c^2 - \frac{5}{2}\right)\right][\varepsilon(z - nh_0c_z) + (1-\varepsilon)l]\Big]\theta(z - nh_0c_z)\theta(l + nh_0c_z - z) - \\ &-\left[\nu + \tau\left(c^2 - \frac{5}{2}\right)\right](1-\varepsilon)l\theta(z - nh_0c_z)\theta(l + (n+1)h_0c_z - z)\Big\} - [\nu + \tau(c^2 - 5/2)]z, \end{aligned} \tag{5}$$

where

$$A = 1 + \frac{2}{3}\frac{\sum\limits_{n=1}^{\infty}(1-\varepsilon)^{n-1}\left\{\left(2\frac{n^2}{l^2} - \frac{l}{n}\right) - \left(\frac{2n}{l} - \frac{l}{n}\right)\left(1 + \frac{n^2}{l^2}\right)^{1/2}\right\}}{\sum\limits_{n=1}^{\infty}(1-\varepsilon)^{n-1}\left\{\frac{n}{l}\left(1 + \frac{n^2}{l^2}\right)^{1/2} - \frac{n^3}{l^3}\ln\left(\frac{l}{n} + \left(1 + \frac{l^2}{n^2}\right)^{1/2}\right)\right\}}.$$

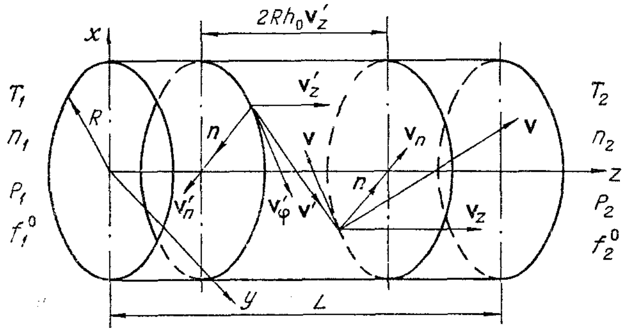

Fig. 1. Geometry of the problem.

To obtain a sufficiently simple analytical solution of integral Eq. (4) with a Cercignani–Lampis scattering core [4], the latter was subjected to certain simplifying transformations. In particular, the following conditions were imposed on the core parameters α_t and α_n corresponding to the accommodation coefficients of the tangential component of momentum and that part of the kinetic energy corresponding to motion along a normal to the wall: $\alpha_n = 1$, $(1 - \alpha_t) << 1$. Here, the complex integrand functions can be expanded into series in powers of $(1 - \alpha_t)$. The thus-simplified integral equation for $h^+(z, \mathbf{c})$ is solved with an accuracy to within the terms proportional to the first power of $(1 - \alpha_t)$. In this case, the unknown function can be represented in the form of a sum

$$h^+(z, \mathbf{c}) = \varphi(z) + c_z g(z). \tag{6}$$

Insertion of (6) into the integral equation for $h^+(z, \mathbf{c})$ makes it possible to break this equation down into a aystem of two integral equations for functions of a single variable $\varphi(z)$ and $g(z)$. Analysis of the system shows that $\varphi(z)$ and $g(z)$ satisfy the symmetry conditions

$$\varphi(z) + \varphi(l - z) = 0, \quad g(z) - g(l - z) = 0. \tag{7}$$

If we ignore the terms proportional to the powers of $(1 - \alpha_t)$ greater than the first power, then this system can be reduced to a single integral equation in $\varphi(z)$. At $\alpha_t = 1$, this equation in turn becomes the Clausing equation. It is solved by the Ritz variational method [6]. Proceeding on the basis of symmetry conditions (7) and the form of the absolute term in the given equation, $\varphi(z)$ can be approximated by the function

$$\varphi(z) = B\left(\frac{l}{2} - z\right),$$

where B is the variational constant. In this case, $g(z)$ is determined by simple integration of the expression for $\varphi(z)$.

As a result of solution of the problem using the Cercignani–Lampis core model [4], perturbation function $h^+(z, \mathbf{c})$ takes the form:

$$h^+(z, \mathbf{c}) = \nu\left\{B_P\left(\frac{l}{2} - z\right) - \frac{1-\alpha_t}{\sqrt{\pi}} c_z(d\,[D(z) + D(l - z)] + [Q(z) + Q(l - z)])\right\} - \frac{\tau}{2}\left\{B_T\left(\frac{l}{2} - z\right) - \frac{1-\alpha_t}{\sqrt{\pi}} c_z d\,[D(z) + D(l - z)]\right\}, \tag{8}$$

where

$$Q(z) = z^3(1 + z^2)^{-1/2}\{(1 + z^{-2})E[(1 + z^2)^{-1/2}] - K[(1 + z^2)^{-1/2}]\};$$

$$\Pi(z) = lz^2(1 + z^2)^{-1/2}\left\{\left(1 + \frac{1}{2z^2}\right)E[(1 + z^2)^{-1/2}] - K[(1 + z^2)^{-1/2}]\right\};$$

$$D(z) = \Pi(z) - Q(z);$$

$$B_T = d\left\{1 - 4(1 - \alpha_t)\int_0^l [D(z) + D(l - z)]^2 dz\,(\pi\Delta)^{-1}\right\};$$

$$B_P = B_T - 8(1-\alpha_t)(\pi\Delta)^{-1}\int_0^l Q(z)[D(z)+D(l-z)]\,dz;$$

$$d = 1 + 2l[(2-l^3)-(2-l^2)(1+l^2)^{1/2}](3\Delta)^{-1};$$
$$\Delta = l(1+l^2)^{1/2} - \ln(l+(1+l^2)^{1/2});$$

E(x) and K(x) are complete elliptic integrals of the first and second classes, respectively.

4. The known distribution of the molecules reflected from the channel wall can be used to also calculate macroscopic characteristics of the flow, including heat and particle fluxes and the gas-temperature distribution near the wall.

The particle flux J_N and heat flux J_q through a cross section of the channel located a distance z_0 from the origin of the coordinates are determined from the expressions:

$$J_N = \frac{4\pi R^2}{\beta^2}\int_{-\infty}^{\infty} dz\int c_n f(z, \mathbf{c})\,d\mathbf{c},$$
$$J_q = \frac{4\pi R^2}{\beta^2}\,kT_1\int_{-\infty}^{\infty} dz\int c_n\left(c^2-\frac{5}{2}\right)f(z, \mathbf{c})\,d\mathbf{c}, \qquad (9)$$

where

$$f(z, \mathbf{c}) = \theta(c_n)\{\theta(c_z+(z-z_0)/h_0)[f_1^0\theta(-z)+f^+(z, \mathbf{c})\theta(z)\theta(z_0-z)] -$$
$$-\theta((z_0-z)/h_0-c_z)[f^+(z, \mathbf{c})\theta(z-z_0)\theta(l-z)+f_2^0\theta(z-l)]\}.$$

If we insert into (9) the function $f^+(z, \mathbf{c})$ found for Maxwell mirror-diffuse boundary conditions, we obtain the following formulas:

$$J_N^M = -\frac{1}{4}n_1\pi R^2 v_T l\left(\nu-\frac{\tau}{2}\right)(1+M_1-AM_2),$$
$$J_q^M = \frac{1}{4}n_1\pi R^2 v_T lkT_1\left\{\frac{\nu}{2}(1+M_1-AM_2)-\frac{\tau}{4}[9(1+M_1)-AM_2]\right\}, \qquad (10)$$

where

$$M_i = \varepsilon^2\sum_{n=1}^{\infty}(1-\varepsilon)^{n-1}N_i\left(\frac{l}{n}, \frac{z_0}{n}\right), \quad i = 1, 2;$$
$$N_1(x, y) = \frac{2}{3}\frac{l}{x^2}\{2+y^3-(1+y^2)^{3/2}+(x-y)^3-[1+(x-y)^2]^{3/2}\};$$
$$N_2(x, y) = \frac{2}{3}\frac{l}{x^2}\left\{2-\frac{3}{2}xy^2+y^3-(1+y^2)^{1/2}\left(1-\frac{3}{2}xy+y^2\right)-\right.$$
$$\left.-\frac{3}{2}x(x-y)^2+(x-y)^3-[1+(x-y)^2]^{1/2}\left[1-\frac{3}{2}x(x-y)+(x-y)^2\right]\right\}.$$

The series M_i converge poorly at $\varepsilon \ll 1$ and diverge at $\varepsilon = 0$. However, according to [1], they can be written in the form

$$M_i = N_i(l, z_0) + 2(1-\varepsilon)\left[N_i\left(\frac{l}{2}, \frac{z_0}{2}\right) - N_i(l, z_0)\right] +$$

$$+\sum_{n=2}^{\infty}(1-\varepsilon)^n\left[(n+1)N_i\left(\frac{l}{n+1}, \frac{z_0}{n+1}\right) - 2nN_i\left(\frac{l}{n}, \frac{z_0}{n}\right) + (n-1)N_i\left(\frac{l}{n-1}, \frac{z_0}{n-1}\right)\right].$$

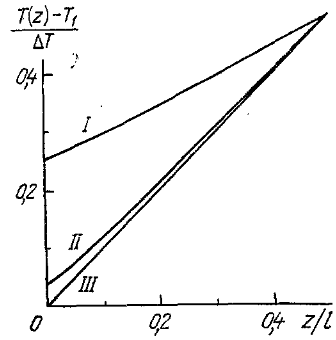

Fig. 2

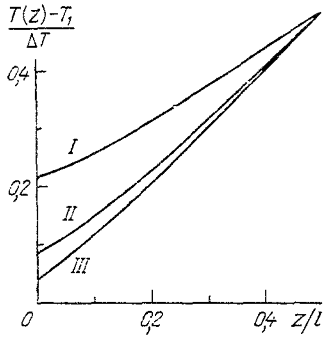

Fig. 3

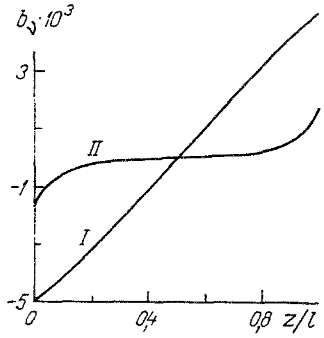

Fig. 4

Fig. 2. The dependence of $(T(z) - T_1)/\Delta T$ on z/l at $\varepsilon = 1$: I) $l = 1$; II) 10; III) 100.

Fig. 3. The dependence of $(T(z) - T_1)/\Delta T$ on z/l at $l = 10$: I) $\varepsilon = 0.1$; II) 0.5; III) 1.0.

Fig. 4. The dependence of b_ν on z/l at $\alpha_t = 0.8$: I) $l = 1$; II) $l = 10$.

In this form, the series converge even at $\varepsilon = 0$. It should be noted that the part of the expression for particle flux J_N^M connected with the pressure gradient at $z_0 = 0, l$ coincides to within the multiplier $n_1 v_T \pi R^2 l\nu/4$ with the expression for the Clausing coefficient obtained by de Marcus [1].

Using the above-indicated simplifications, we can obtain the following for the Cercignani–Lampis scattering core

$$J_N^C = -\frac{1}{2} n_1 R^2 \left(\frac{\pi}{\beta}\right)^{1/2} l \{\nu [1 + N_1(l, z_0) - B_P N_2(l, z_0) + I_1 + I_2] - \tau [1 + N_1(l, z_0) - B_T N_2(l, z_0) + I_1]/2\},$$

$$J_q^C = \frac{1}{2} P_1 R^2 \left(\frac{\pi}{\beta}\right)^{1/2} l \left\{\frac{\nu}{2} [1 + N_1(l, z_0) - B_P N_2(l, z_0)] - \frac{\tau}{4} [9 + 9N_1(l, z_0) - B_T N_2(l, z_0)]\right\}, \tag{11}$$

where

$$I_1 = \frac{4(1-\alpha_t)}{\pi l^2} d \int_0^l \Pi(z - z_0) [D(z) + D(l - z)] dz;$$

$$I_2 = \frac{4(1-\alpha_t)}{\pi l^2} \int_0^l \Pi(z - z_0) [Q(z) + Q(l - z)] dz.$$

Since the channel walls are impermeable, chemical reactions are absent, and we are dealing with a steady regime of gas motion, the dependence of the particle flux on the longitudinal coordinate z is a consequence of the approximate nature of the solution of the integral equations for the perturbation function – in particular, the approximation of the solutions by linear functions of z. It was rigorously proven in [8] that for a free-molecular gas flow in a long channel, such an approximation makes it possible to calculate the flows in the middle of the channel to within terms on the order of $o(l^{-1} \ln l)$.

The asymptotic expressions for J_N^M and J_q^M in the approximation $l^{-1}\ln l \ll \varepsilon$ and for J_N^C and J_q^C in the approximation $l^{-1}\ln l \ll 1$, found from general formulas, will be as follows at $z_0 = 0, l$

$$J_N^M = -\frac{1}{4} n_1 v_T \pi R^2 \frac{4}{3} \frac{2-\varepsilon}{\varepsilon} \left(\nu - \frac{\tau}{2}\right),$$

$$J_q^M = \frac{1}{4} P_1 v_T \pi R^2 \frac{4}{3} \frac{2-\varepsilon}{\varepsilon} \left(\frac{\nu}{2} - \frac{5}{4}\tau \right),$$

$$J_N^C = -\frac{1}{4} n_1 v_T \pi R^2 \frac{4}{3} \left\{ \nu \left[1 + \frac{3\pi}{8}(1-\alpha_t) \right] - \frac{\tau}{2} \right\}, \tag{12}$$

$$J_q^C = \frac{1}{4} P_1 v_T \pi R^2 \frac{4}{3} \left\{ \frac{\nu}{2} \left[1 + \frac{3\pi}{64}(1-\alpha_t) \left(5 - \frac{3\pi}{10} \right) \right] - \frac{5}{4}\tau \left[1 - \frac{9\pi^2}{3200}(1-\alpha_t) \right] \right\}.$$

At $z_0 = l/2$, the particle fluxes remain the same as at $z_0 = 0$, l, while the heat fluxes take the form

$$J_q^M = \frac{1}{4} P_1 v_T \pi R^2 \frac{4}{3} \frac{2-\varepsilon}{\varepsilon} \left(\frac{\nu}{2} - \frac{9}{4}\tau \right),$$

$$J_q^C = \frac{1}{4} P_1 v_T \pi R^2 \frac{4}{3} \left(\frac{\nu}{2} - \frac{9}{4}\tau \right). \tag{13}$$

There is no longer any physical validity to the dependence on the longitudinal coordinate z in the asymptotic expressions for J_N^M and J_N^C, while at the same time the heat flows remain functions of z. In the flow J_q^M along the channel, only that part due to the temperature gradient changes. In J_q^C, the parts which change are connected with the temperature gradient and the pressure gradient. This difference in the heat fluxes in the middle and at the ends of the channel can only be attributed to the presence of heat flows through the lateral surfaces of both halves of the channel which are equal in absolute value but opposite in sign.

The expressions for the heat flux through the lateral surface of the left half have the form:

$$_{\frac{1}{2}}J_q^M = -\frac{1}{4} P_1 v_T \pi R^2 \frac{4}{3} \frac{2-\varepsilon}{\varepsilon} \tau,$$

$$_{\frac{1}{2}}J_q^C = -\frac{1}{4} P_1 v_T \pi R^2 \frac{4}{3} \left\{ \nu \frac{3\pi}{128} \left(5 - \frac{3\pi}{10} \right)(1-\alpha_t) + \tau \left[1 + \frac{45\pi^2}{12800}(1-\alpha_t) \right] \right\}. \tag{14}$$

However, the heat flux through the entire lateral surface is equal to zero, since the density of the flux is antisymmetrical relative to the middle of the channel. In connection with the presence of heat fluxes normal to the channel wall, it is interesting to calculate the gas-temperature distribution near the surface of the solid T(z). It is found in a linear approximation on the basis of a kinetic determination of temperature [9]:

$$T^M(z) = T_1 \left\{ 1 + \frac{\tau}{2} \left[l + \frac{2}{\pi} \varepsilon \sum_{n=1}^{\infty} (1-\varepsilon)^{n-1} \quad ((n^2 + z^2)^{1/2} E\,[n(n^2+z^2)^{-1/2}] - \right.\right.$$

$$\left.\left. - (n^2 + (l-z)^2)^{1/2} E\,[n(n^2 + (l-z)^2)^{-1/2}\,]) \right] \right\}, \tag{15}$$

$$T^C(z) = T_1 \{ 1 - \nu l b_\nu + \tau l b_\tau \},$$

where

$$b_\nu(z) = \frac{1-\alpha_t}{6\pi l} \int_0^l \operatorname{sign}(z-y)\,[1 + (z-y)^2]^{-3/2} \{ Q(y) + Q(l-y) + d\,[D(y) + D(l-y)] \}\,dy,$$

$$b_\tau(z) = \frac{1}{2} \left\{ 1 + \frac{2}{\pi l} [(1+z^2)^{1/2} E\,[(1+z^2)^{-1/2}] - (1 + (l-z)^2)^{1/2} E\,[(1 + (l-z)^2)^{-1/2}]] + \right.$$

$$\left. + \frac{d}{6\pi l}(1-\alpha_t) \int_0^l \operatorname{sign}(z-y)\,[1 + (z-y)^2]^{-3/2} [D(y) + D(l-y)]\,dy \right\}.$$

5. The distributions of gas temperature near the wall obtained here (15) differ from the linear temperature distribution for the channel surface. This difference is also connected with the existence of heat flows directed along normals to the side of the channel. Figures 2 and 3 show the temperature distribution $T^M(z)$ in the case of mirror-diffuse boundary condi-

tions. It is evident from the data that the difference between the kinetic temperature of the gas and the temperature of the channel surface is greatest at the ends of the channel. This difference increases both with a reduction in channel length and with a reduction in the fraction of diffuse reflection, i.e., l and ε have a similar effect on the distribution of the kinetic temperature of the gas in the channel.

In contrast to the solution obtained with mirror-diffuse boundary conditions, the solution obtained on the basis of the Cercignani—Lampis scattering core model makes it possible to conclude that the heat flux density normal to the wall may be nontrivial even in the absence of a temperature difference at the ends of the channel. Figure 4 shows the dependence of the function b_ν on the ratio z/l, characterizing the temperature distribution near the channel wall with the motion of the gas under the influence of a pressure gradient. The contribution to b_T connected with the deviation of α_t from 1 is small (about 0.2%) in the given solution, so the graph of this function in relation to the ratio z/l nearly coincides with the curve III in Fig. 3. It follows from Figs. 2-4 that the transverse heat flows are localized at the ends of the channel.

NOTATION

$c = (c_n, c_\varphi, c_z)$, dimensionless velocity of the gas molecules; f_1°, f_2°, Maxwell distributions of the gas molecules in the volumes; f^-, f^+, distributions of the gas molecules incident on the channel wall and reflected from the wall; h^+, perturbation function; J_N, J_q, mass and heat fluxes through the cross section of the channel; ${}_{1/2}J_q$, heat flux through half of the lateral surface of the channel; k, Boltzmann constant; L, channel length; l, dimensionless channel length; m, mass of a gas molecule; n, numerical density; P, pressure; R, channel radius; T, temperature; $\theta(x)$, Heaviside function; v', v, absolute velocities of the gas molecules incident upon and reflected from the channel wall; v_T, mean thermal velocity of the gas molecules; $W(z, c' \to c)$, scattering core; z, longitudinal coordinate; ε, fraction of gas molecules diffusively reflected by the wall.

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